Math 2FM3, Tutorial 4

Oct 4th, 2015

Annuity-Due

- Payments are made at the beginning of the period.

- Accumulated Value:
- $\ddot{s}_{n|i} = (1+i)^n + \ldots + (1+i) = (1+i)\, s_{n|i} = [(1+i)^n - 1]/d$
- Present Value:
- $\ddot{a}_{n|i} = 1 + v + \ldots + v^{n-1} = (1 - v^n)/d$

Perpetuities

- The infinite period annuity (n goes to infinity) is called a perpetuity.
- $a_{\infty| i} = \lim_{n\to\infty} (1-v^n)/i=1/i$
- $\ddot{a}_{\infty|i} = \lim_{n\to\infty}(1-v^n)/d =1/d$

Ex 2.2.5

- A perpetuity paying 1 at the beginning of each 6-month period has a present value of 20. A second perpetuity pays X at the beginning of every 2 years. Assuming the same effective annual interest rate, the two present values are equal. Determine X.

- j is 6-month interest rate
- $d_j = j/(1+j)$ is discount
- Since the present value $1 * \ddot{a}_{\infty|i} = 20$, then we get $1/d_j = 20$, then $j = 1/19$.
- The 2-year rate $i = (1+j)^4 - 1 = (1+1/19)^4 - 1$
- Hence, by $X * \ddot{a}_{\infty|i} = 20$, we can obtain $X = 20d_i = 20i/(1+i) = 20 * [(1+1/19)^4 - 1]/(1+1/19)^4 = 3.71$.

Ex 2.2.6

- Sally lends 10,000 to Tim. Tim agrees to pay back the loan over 5 years with monthly payments at the end of each month. Sally can reinvest the payments from Tim in a saving account paying interest at 6%, compounded monthly. The yield rate earned on Sally's investment over the five-year period turned out to be 7.45%, compounded semi-annually. What nominal rate of interest, compounded monthly, did Sally charge Tim on the loan?

Ex 2.2.9

- On the first day of every January, April, July and October Smith deposits 100 in an account earning $i^{(4)} = 0.16$. He continues the deposits until he accumulates a sufficient balance to begin withdrawals of 200 every 3 months, starting 3 months after the final deposit, such that he can make twice as many withdrawals as he made deposits. How many deposits are needed?